Before we begin describing motion, we must first differentiate between a **scalar** and a **vector** quantity.

A **vector quantity** requires *both* a direction and a magnitude (size of the number) to describe. While a **scalar quantity** is described *only* by magnitude.

For example, the velocity (a vector) of a track runner may be described as “13 m/s toward the finish line” while her speed (a scalar) would be described as just “13 m/s.”

<table>
<thead>
<tr>
<th>A Few Examples of Vectors</th>
<th>A Few Examples of Scalars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement, Velocity, Acceleration, Force, Momentum, Torque,</td>
<td>Mass, Volume, Density, Temperature, Speed,</td>
</tr>
<tr>
<td>Fields (Gravitational, Electric, and Magnetic).</td>
<td>Energy, Efficiency</td>
</tr>
</tbody>
</table>

Scalar quantities are typically represented as positive numbers. Vectors quantities may be positive or negative numbers since the mathematical sign of the number tells you the direction that the vector is moving. Below is the common convention used in Physics:

- Up/Right: Positive
- Down/Left: Negative

When describing motion, there are five main quantities that concern us:

1. Position (m)
2. Displacement (m)
3. Velocity/Speed (m/s)
4. Acceleration (m/s²)
5. Time (s)

We will focus on defining these quantities and interpreting them graphically in situations in which an object only moves in one dimension – either Up/Down or Right/Left.

### I. Position

Position is defined as a **specific location** and is typically represented with the symbol \( x \). The symbol is often followed by a subscript to differentiate between two or more positions.

- SI Unit: m
- \( x_f \) represents the final position (ending point), \( x_i \) or \( x_0 \) represents the initial position (starting point), and \( x_1, x_2, x_3, etc. \) represents an arbitrary position in between the initial and final position.
- Scalar Quantity

It is useful to have a visual representation of an object’s position. Typically we use the Cartesian coordinate system, like you’ve used in your math courses. For 1D motion, our coordinate system will essentially be a number line.

\[
\begin{array}{ccccccccccc}
-3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
x_0 & x_1 & x_3 & x_f
\end{array}
\]

The number value of each tick mark is usually determined by the problem or by the person studying the problem.
II. Displacement

Displacement is simply defined as the change in an object’s position and is typically represented with the symbol $\Delta x$ (The Greek letter delta, $\Delta$, is used to represent a change in a quantity). Displacement is not the same as distance traveled. Consider an example.

You run 20 laps on the track, which is approximately 8,000 m – your distance traveled. However, since you started and stopped at the same point, there is no net change in your position making your displacement 0 m!

Symbolically, displacement is defined as:

$$\text{Displacement} \quad \Delta x = x_f - x_i$$

- SI Unit: m
- $x_f$ represents the final position and $x_i$ represents the initial position.
- Vector Quantity

Displacement is sometimes called a state function, meaning only the final and initial quantities matter. What happens in the middle is of no concern. You could run 1 lap or 300 laps on the track, but your displacement is zero for both because your starting and stopping positions are identical in both cases!

Graphical Representation

It is often useful to measure an object’s position at various times and to create a graph from the data. The graph to the right is an example of a position vs. time graph.

Note: When saying we plot one variable vs. another, it is understood to be (dependent variable) vs. (independent variable). Or you can think of it as (vertical axis) vs. (horizontal axis).

To find the displacement over a given time interval, you would simply subtract the vertical coordinates (position) at the two times of interest.

III. Velocity/Speed

As already noted, velocity is not the same as speed. To further distinguish the two, let’s look at their respective definitions.

The average speed of an object over some time interval is given by:

$$\text{Average Speed} \quad \frac{\text{total distance}}{\text{total time}}$$

- SI Unit: $\frac{\text{m}}{\text{s}}$
- Scalar quantity

The average speed is defined as how much distance an object has traveled in a time interval.
The average velocity on the other hand, depends on *displacement* rather than *total distance*. Make sure you keep the distinction sharp in your mind. The average velocity is the displacement divided by total time.

**Average Velocity**

\[
\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0} \quad \text{(if } t_0 = 0.0 \text{ s)}
\]

- SI Unit: \( \frac{m}{s} \)
- Vector quantity
- \( \bar{v} \) is pronounced “v bar” and the bar indicates it is an average quantity.
- The \( \equiv \) symbol signifies the equation is also a definition. It is not necessary to use it in your work.

The average velocity is defined by how much displacement an object has traveled in a time interval.

For example, look at the graph below. Let’s calculate the average velocity between various points:

![Graph showing motion](image)

**Average Velocity between O and A**

\[
\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2 \text{ m} - 0 \text{ m}}{4 \text{ s} - 0 \text{ s}} = \frac{2 \text{ m}}{4 \text{ s}} = 0.5 \frac{\text{m}}{\text{s}}
\]

**Average Velocity between A and B**

\[
\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ m} - 2 \text{ m}}{8 \text{ s} - 4 \text{ s}} = \frac{8 \text{ m}}{4 \text{ s}} = 2 \frac{\text{m}}{\text{s}}
\]

**Average Velocity between B and C**

\[
\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0 \text{ m} - 10 \text{ m}}{12 \text{ s} - 8 \text{ s}} = \frac{-10 \text{ m}}{4 \text{ s}} = -2.5 \frac{\text{m}}{\text{s}}
\]

Sometimes, we aren’t interested in the average velocity over a time interval. Instead, we may want the velocity at an *exact instant* in time. To achieve this, we simply shrink the time interval to be really really really small (we call it “approaching zero” or “infinitesimal”). We show “approaching zero” with the following notation:

\[
\lim_{\Delta t \to 0}
\]

Let’s Illustrate:

Symbolically we define instantaneous velocity as:

**Instantaneous Velocity**

\[
\Delta v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}
\]

- SI Unit: \( \frac{m}{s} \)
- Vector quantity
**Graphical Representation**

The definition of average velocity, $\bar{v} = \frac{\Delta x}{\Delta t}$, should strike a familiar chord in your algebraic brain. It looks a lot like the slope formula: $m = \frac{\Delta y}{\Delta x}$.

Graphically speaking, the average velocity of an object is the slope between two points on the curve of a displacement vs. time graph or a position vs. time graph.

If you look at the illustrated example above, you might see that the instantaneous velocity is equal to the slope of the line tangent to the curve at time $t$ in question. In other words, $v_t = m_{\text{tangent, } t}$.

In case you forgot what a tangent line is, it is a line that touches a curve once and only once at a given point. If the curve is a line, then the instantaneous and average values are the same along the entire length of the line. Can you think of why this is true?

It is often useful to graph an object’s velocity at various times. An example is given to the left.

If you look at the units, you may notice that we can relate a velocity vs. time graph to the displacement of an object.

When we multiply the two units, we have: $\frac{m}{s} \cdot s = m$.

**This indicates that the net (total) area between a velocity vs. time graph and the t-axis gives the displacement over the time interval.** If you don’t see how, just ask and I’ll be glad to show you.

**Note:** Area above the t-axis is considered positive displacement and the area under the t-axis is considered negative displacement.

Let’s find the displacement of the moving object between 0.0 s and 2.0 s. Find the area bounded by the curve and the time axis. Notice that the area is a triangle; find its area.

**Displacement from 0.0 s to 2.0 s**

$A = \frac{1}{2} bh = \frac{1}{2} (2.0 \, s) \left( 4.0 \, \frac{m}{s} \right) = 4.0 \, m$ : The object traveled 4.0 m in the positive direction
Let's also find the displacement from 0.0 s to 3.0 s. Notice that from 0.0 s to 2.0 s, the area is the triangle we calculated above (4.0 m). We must now add that to the remaining area; the area of a rectangle from 2.0 s to 3.0 s.

Displacement from 2.0 s to 3.0 s
\[
A = bh = (1.0 \text{ s}) \left( 4.0 \text{ m/s} \right) = 4.0 \text{ m}
\]

In total, the object traveled 4.0 m in the positive direction from 0.0 s to 2.0 s and 4.0 m in the positive direction from 2.0 s to 3.0 s, which is a total of 8.0 m in the positive direction. In equation form, we’d write: \(\Delta x_{0,2} + \Delta x_{2,3} = 4.0m + 4.0m = 8.0m\).

**IV. Acceleration**

Acceleration is defined as the rate of change of velocity. So, any time an object’s speed or direction changes, it is accelerating! If, while driving, you take a curve at a constant speed, you are still accelerating because your direction is changing. If you increase speed or decrease speed, you are accelerating since the speed is changing.

The average acceleration over a time interval is defined as:

\[
\overline{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0} = \frac{v_f - v_0}{t} \quad \text{(if } t_0 = 0.0 \text{ s)}
\]

- SI Unit: \( \frac{m}{s^2} \)
- Vector

If we were to look at a velocity vs. time graph, we could come up with a definition for instantaneous acceleration in the same manner we came up with a definition for instantaneous velocity. There’s no need to repeat this process twice.

**Instantaneous Acceleration**

\[
a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}
\]

- SI Unit: \( \frac{m}{s^2} \)
- Vector

**Graphical Representation**

Again, you may notice that the definition of average acceleration looks a lot like the slope formula.

**Graphically speaking, the average acceleration of an object is the slope between two points on the curve of a velocity vs. time graph.** The instantaneous acceleration is the slope of the tangent line at time \( t \) on a curve on a velocity vs. time graph.
For example, look at the graph below. Let’s calculate the average acceleration between various points:

\[\text{Average Acceleration between } O \text{ and } A\]
\[\bar{a} = \frac{\Delta v}{\Delta t} = \frac{2 \text{ m/s} - 0 \text{ m/s}}{1 \text{ s} - 0 \text{ s}} = \frac{2 \text{ m}}{1 \text{ s}} = 2 \frac{\text{ m}}{\text{ s}^2}\]

\[\text{Average Acceleration between } A \text{ and } B\]
\[\bar{a} = \frac{\Delta v}{\Delta t} = \frac{4 \text{ m/s} - 2 \text{ m/s}}{2.5 \text{ s} - 1 \text{ s}} = \frac{2 \text{ m/s}}{1.5 \text{ s}} = 1.33 \frac{\text{ m}}{\text{ s}^2}\]

\[\text{Average Acceleration between } B \text{ and } C\]
\[\bar{a} = \frac{\Delta v}{\Delta t} = \frac{3 \text{ m/s} - 4 \text{ m/s}}{3.5 \text{ s} - 2.5 \text{ s}} = \frac{-1 \text{ m/s}}{1 \text{ s}} = -1 \frac{\text{ m}}{\text{ s}^2}\]

The net area under an acceleration vs. time curve gives us the final velocity of the object. Again area above the t-axis is taken to be positive velocity (traveling up/right), while area under the t-axis is taken to be negative velocity (traveling down/left).

**V. Time**

Defining and analyzing time in physics is actually a very complex problem thanks to Albert Einstein and friends. However, for this class, we will simply consider time to be the dimension that requires events to unfold in a unidirectional and ordered sequence of events. The SI unit for time is the second, which is defined as 9,192,631,700 times the period of oscillation of radiation from a Cesium atom.

**Summary/Tips**

It may prove useful to summarize the above information.

<table>
<thead>
<tr>
<th>Type of Graph</th>
<th>Slope</th>
<th>Tangent</th>
<th>Area Underneath</th>
</tr>
</thead>
<tbody>
<tr>
<td>x vs. t</td>
<td>(\bar{v})</td>
<td>(v)</td>
<td>-</td>
</tr>
<tr>
<td>v vs. t</td>
<td>(\bar{a})</td>
<td>(a)</td>
<td>Net Displacement</td>
</tr>
<tr>
<td>a vs. t</td>
<td>-</td>
<td>-</td>
<td>Change In Velocity</td>
</tr>
</tbody>
</table>

Remember: \(\Delta x = x_f - x_i\) \(\bar{v} = \frac{\Delta x}{\Delta t}\) \(\bar{a} = \frac{\Delta v}{\Delta t}\)

If you forget what the slope or area represents on a given graph, then look at the units of each axis! It is imperative that you always look at the labels of each axis!! Don’t assume the graph is \(x \text{ vs. } t\) when it might be \(v \text{ vs. } t\), etc.

One final thing to keep in mind: If a value is held constant over time, then the average and instantaneous values are always the same! Can you think of why this is true? Think about trial 1 or trial 2 on our “Walker Lab.” If you can’t figure it out, be sure to ask in class!
Examples
Let’s consider a few graphs and note some key ideas together. Then we’ll work on solving some problems.
## Problems

1. Below is a velocity (m/s) vs. time (s) graph for an object moving horizontally in one dimension. For each time interval, explain what is happening to the object's (a) Velocity, (b) Speed, (c) Acceleration, and (d) Displacement.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Velocity</th>
<th>Speed</th>
<th>Acceleration</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – A</td>
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<td></td>
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<tr>
<td>A - B</td>
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<tr>
<td>B - C</td>
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<tr>
<td>C - D</td>
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<tr>
<td>D - E</td>
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<tr>
<td>E - F</td>
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<tr>
<td>F - G</td>
<td></td>
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</tbody>
</table>

2. Below is a velocity (m/s) vs. time (s) graph for an object moving horizontally in one dimension. For each time interval, explain what is happening to the object’s (a) Velocity, (b) Speed, (c) Acceleration, and (d) Displacement.

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<tr>
<th>Interval</th>
<th>Velocity</th>
<th>Speed</th>
<th>Acceleration</th>
<th>Displacement</th>
</tr>
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<tbody>
<tr>
<td>0 – A</td>
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<tr>
<td>A - B</td>
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<td>B - C</td>
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<td>C - D</td>
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<td>D - E</td>
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<td>E - F</td>
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<tr>
<td>F - G</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Use the graph to find the average velocity from:
(a) 0 s to 2 s,
(b) 2 s to 4 s,
(c) 4 s to 5 s,
(d) 5 s to 7 s,
(e) 7 s to 8 s,
(f) 0 s to 8 s
(g) On the grid below, plot the data points for the quantities you have identified in part (a)-(g) to construct a corresponding velocity (m/s) vs. time (s) graph. Be sure to label your axes and show the scale that you have chosen for the graph.

(h) Determine each time value in which the object changes its direction.
4. A drag racer accelerates rapidly to its top speed and then maintains this speed until it reaches the finish line. The racer then deploys its parachute and hits the brakes to bring the car to a rapid stop. Draw plausible (a) $x$ vs. $t$, (b) $v$ vs. $t$, and (c) $a$ vs. $t$ graphs.

5. Use the velocity vs. time graph to calculate the instantaneous acceleration at:
   (a) 2 s
   (b) 10 s
   (c) 18 s
   (d) Determine the net displacement of the object.
   (e) Construct the corresponding position vs. time graph.
   (f) Construct the corresponding acceleration vs. time graph.
6. A 0.50 kg cart moves on a straight horizontal track. The graph of velocity (v) versus time (t) for the cart is given below.

(a) Indicate every time for which the cart is at rest.

(b) Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

(c) Determine the horizontal position x of the cart at t = 9.0 s if the cart is located at x = 2.0 m at t = 0.

(d) Determine each time value in which the 0.5 kg object changes its direction.
(e) On the axes below, sketch the acceleration \( a \) versus time \( t \) graph for the motion of the cart from \( t = 0 \) to \( t = 25 \) s.

(f) Explain how the graph above would change if the initial acceleration of the 0.5 kg object was \(-4.0\) m/s\(^2\). HINT: There is more than one change in the graph.

(g) Assume that the original velocity vs. time graph does not contain a straight line from \( t = 17.0 \) s to \( t = 20.0 \) s, but contains a parabolic curve that begins with the same initial velocity and ends with the same final velocity. Explain what this means for the motion of the 0.5 kg object.
7. The vertical position of an elevator as a function of time is shown below.

(a) Explain what is physically happening to the elevator from \( t = 0 \) s to \( t = 8 \) s, \( t = 8 \) s to \( t = 10 \) s, \( t = 10 \) s to \( t = 18 \) s, \( t = 18 \) s to \( t = 20 \) s, and \( t = 20 \) s to \( t = 25 \) s.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0s – 8s</td>
<td></td>
</tr>
<tr>
<td>8s – 10s</td>
<td></td>
</tr>
<tr>
<td>10s – 18s</td>
<td></td>
</tr>
<tr>
<td>18s – 20s</td>
<td></td>
</tr>
<tr>
<td>20s – 25s</td>
<td></td>
</tr>
</tbody>
</table>

(b) On the grid below, graph the velocity of the elevator as a function of time. Indicate where there is a zero acceleration, a nonzero acceleration, and where the elevator changes direction.
8. A horse canters away from its trainer in a straight line, moving 116 m away in 14.0 s. It then turns abruptly and gallops halfway back in 4.8 s. (a) Determine the horse’s average speed. (b) Determine the horse’s average velocity. (c) Using the formulas for average speed and average velocity, explain how the two quantities would change if the horse stood still for 5.0 s after the initial 116 m traveled distance.

9. A car traveling 25 m/s is 110 m behind a truck that is traveling 20 m/s. (a) Determine how long will it take for the car to catch up to the truck. (b) Suppose part (a) was changed so that the car was traveling at 30 m/s at a distance of 130 m behind the truck that is traveling 20 m/s, explain how could you determine the position of the car relative to the truck (behind, next to, in front of) after using the time that was found in part (a).

10. A sports car moving at a constant speed travels 110 m in 5.0 s. The car then brakes and comes to a stop in 4.0 s. (a) Determine the acceleration of the car. (b) If someone were to ask the position of the car after 2.0 s, explain how you could use the information above to answer the person’s question. (c) Explain how the stopping distance of the car compares to the original stopping distance if the car’s acceleration from part (a) is increased.

11. An airplane travels 3100 km at a speed of 790 km/hr and then encounters a tailwind that boosts its speed to 990 km/hr for the next 2800 km. (a) Determine the total time for the trip? (b) Determine the average speed of the plane for this trip.
Spreadsheet Project
Use spreadsheet software to finish the following problem.

12. The position of a racing car, which starts from rest at $t = 0.00$ s and moves in a straight line, is given as a function of time in the following Table. (a) Complete the value of the car’s velocity at each time interval. (b) Complete the car’s acceleration at each time interval. (c) Create plots of $x$ vs. $t$, $v$ vs. $t$, and $a$ vs. $t$.

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>0.00</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
<th>3.50</th>
<th>4.00</th>
<th>4.50</th>
<th>5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ (m)</td>
<td>0.00</td>
<td>0.11</td>
<td>0.46</td>
<td>1.06</td>
<td>1.94</td>
<td>4.62</td>
<td>8.55</td>
<td>13.79</td>
<td>20.36</td>
<td>28.31</td>
<td>37.65</td>
<td>48.37</td>
<td>60.30</td>
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<tr>
<td>$v$ (m/s)</td>
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<tr>
<td>$a$ (m/s$^2$)</td>
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</tbody>
</table>
### Problem Solutions

1. | Interval | Velocity | Speed | Acceleration (all are constant) | Displacement |
---|---|---|---|---|
0 – A | Increasing at a constant rate | Increasing at a constant rate | Positive | Increasing |
A - B | Increasing at a constant rate | Increasing at a constant rate | Positive | |
B - C | Constant and positive | Constant | Zero | |
C - D | Decreasing at a constant rate | Decreasing at a constant rate | Negative | |
D - E | Decreasing at a constant rate | Increasing at a constant rate | Negative | Decreasing |
E - F | Constant and negative | Constant | Zero | |
F - G | Increasing at a constant rate | Decreasing at a constant rate | Positive | |

2. | Interval | Velocity | Speed | Acceleration (all are constant) | Displacement |
---|---|---|---|---|
0 – A | Zero | Zero | Zero | Zero |
A - B | Zero | Zero | Zero | Zero |
B - C | Suddenly positive then decreasing at a constant rate | Suddenly positive then decreasing at a constant rate | Negative | Increasing |
C - D | Zero then decreasing at a constant rate | Zero then increasing | Negative | Decreasing |
D - E | Zero | Zero | Zero | Zero |
E - F | Increasing at a constant rate | Increasing at a constant rate | Positive | Increasing |
F - G | Constant and positive | Constant and positive | Zero | Increasing |
3. Use the graph to find the average velocity from:
   a) 0s to 2 s
   b) 2s to 4s
   c) 4s to 5s
   d) 5s to 7s
   e) 7s to 8s
   f) 0s to 8s
   g) Construct a corresponding v vs. t graph

   \[
   a) \quad \overline{V} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ m}}{2 \text{ s}} = 5 \frac{\text{ m}}{\text{ s}} \\
   b) \quad \overline{V} = \frac{5 \text{ m} - 10 \text{ m}}{2 \text{ s}} = -2.5 \frac{\text{ m}}{\text{ s}} \\
   c) \quad \overline{V} = \frac{-5 \text{ m} - 5 \text{ m}}{2 \text{ s}} = -5 \frac{\text{ m}}{\text{ s}} \\
   d) \quad \overline{V} = \frac{0 \text{ m} - 0 \text{ m}}{8 \text{ s}} = 0 \frac{\text{ m}}{\text{ s}} \\
   
   \]

4. A drag racer accelerates rapidly to its top speed and then maintains this speed until it reaches the finish line. The racer then deploys its parachute and hits the brakes bring the car to a rapid stop. Draw plausible (a) x vs. t, (b) v vs. t, and (c) a vs. t graphs.
5. Use the $v$ vs. $t$ graph to do calculate the instantaneous acceleration at:

a. 2 s  

b. 10 s  

c. 18 s  

d. What is the net displacement of the object?  

e. Construct the corresponding $x$ vs. $t$ graph

\[ a = 0 \text{ m/s}^2 \]

\[ a_{10} = \frac{\Delta v}{\Delta t} = \frac{8 \text{ m/s} - (-8 \text{ m/s})}{15 \text{ s} - 5 \text{ s}} = 1.6 \text{ m/s}^2 \]

\[ a = 0 \text{ m/s}^2 \]

[Area calculations]

\[ \Delta x = \text{Area} = 5s(-8 \frac{m}{s}) + \frac{1}{2}(5s)(-8 \frac{m}{s}) + \frac{1}{2}(5s)(8 \frac{m}{s}) + 5s(8 \frac{m}{s}) = 0 \text{ m} \]

[Assuming we start at $x = 0$ → a quick sketch]
6.

a) \( v = 0 \text{ m/s at } t = 4 \text{s, } 18 \text{s} \)

b) \((4,9), (18,20)\)

c) \( \Delta x = 2 + \text{Area}_{0,4} - \text{Area}_{4,9} = 2 + \left( \frac{1}{2} \right) (0.8)(4) - \left( \frac{1}{2} \right) (1)(5) = 1.1 \text{ m} \)

d) \( v \) changes sign at 4s and 18 s

e) Horizontal line segments at heights that correspond to the slopes of the various portions of the graph.

f) (I am unsure of the intent of this question. Email Kyle)

g) The object accelerated from the initial to the final velocity (for this interval) at an increasing rate instead of a constant rate.

7.

a/b)

<table>
<thead>
<tr>
<th>Interval</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0s – 8s</td>
<td>The elevator is rising with a constant velocity. The ( v ) vs. ( t ) graph will be horizontal and positive at ( 12/8 = 1.5 \text{ m/s} )</td>
</tr>
<tr>
<td>8s – 10s</td>
<td>The elevator accelerates negatively until it comes to rest. The ( v ) vs. ( t ) graph will curve and approach zero.</td>
</tr>
<tr>
<td>10s – 18s</td>
<td>The elevator remains at rest at a height of 14 m. The ( v ) vs. ( t ) graph will be horizontal and zero.</td>
</tr>
<tr>
<td>18s – 20s</td>
<td>The elevator accelerates negatively, speeding up in the downward direction. The ( v ) vs. ( t ) graph will curve and approach (-12/6 = -2 \text{ m/s})</td>
</tr>
<tr>
<td>20s – 25s</td>
<td>The elevator descends to the ground-level with a constant velocity. The ( v ) vs. ( t ) graph will be horizontal and negative at (-2 \text{ m/s})</td>
</tr>
</tbody>
</table>

8.

a) \( \bar{s} = \frac{(116m + 58m)}{14.0s + 4.8s} = 9.255 \frac{m}{s} \)

b) \( \bar{v} = \frac{116m - 58m}{18.8s} = 3.085 \text{ m/s} \)

c) Both quantities would decrease because the distance and displacement would both be the same but the total time would increase.

9.

a) \((25 \text{ m/s} - 20 \text{ m/s}) = 5 \text{ m/s} \rightarrow (110 \text{ m})/(5 \text{ m/s}) = 22 \text{ s}\)

b) \( \Delta x = \left( 10 \frac{m}{s} \right) (22 \text{ s}) = 220 \text{ m} > \text{130m} \Rightarrow \text{The car will be in front of the truck.}\)

10.

a) \( v = \frac{110 \text{ m}}{5s} = 22 \frac{\text{m}}{s} \Rightarrow a = \frac{22 \text{ m}}{4s} = 5.5 \frac{\text{m}}{s^2} \)

b) Since the speed is constant for the first 5 seconds and we know that speed, we could use \( \Delta x = vt \) to find the car’s position.

c) The stopping distance will be decreased because the car will come to rest in a shorter amount of time.
11.

a) \( \frac{3100 \text{ km}}{790 \text{ km/hr}} + \frac{2800 \text{ km}}{990 \text{ km/hr}} = 6.752 \text{ hr} \)

b) \( \bar{s} = \frac{5900 \text{ km}}{6.752 \text{ hr}} = 873.382 \text{ km/hr} \)

12. Using the fact that \( \bar{v} = \frac{\Delta x}{\Delta t} \) and \( \bar{a} = \frac{\Delta v}{\Delta t} \), a spreadsheet allows us to calculate the \( v \) and \( a \) values and then create a graph. These points should technically be plotted at the midpoints of the intervals, but this is a technical issue that we’ll discuss later.

<table>
<thead>
<tr>
<th>t (s)</th>
<th>0.00</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
<th>3.50</th>
<th>4.00</th>
<th>4.50</th>
<th>5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>x (m)</td>
<td>0.00</td>
<td>0.11</td>
<td>0.46</td>
<td>1.06</td>
<td>1.94</td>
<td>4.62</td>
<td>8.55</td>
<td>13.79</td>
<td>20.36</td>
<td>28.31</td>
<td>37.65</td>
<td>48.37</td>
<td>60.30</td>
</tr>
<tr>
<td>v (m/s)</td>
<td>-</td>
<td>0.44</td>
<td>1.40</td>
<td>2.40</td>
<td>3.52</td>
<td>5.36</td>
<td>7.86</td>
<td>10.48</td>
<td>13.14</td>
<td>15.90</td>
<td>18.68</td>
<td>21.44</td>
<td>23.86</td>
</tr>
<tr>
<td>a (m/s²)</td>
<td>-</td>
<td>-</td>
<td>3.84</td>
<td>4.00</td>
<td>4.48</td>
<td>3.68</td>
<td>5.00</td>
<td>5.24</td>
<td>5.32</td>
<td>5.52</td>
<td>5.56</td>
<td>5.52</td>
<td>4.84</td>
</tr>
</tbody>
</table>