Bouncing, Regression, and Analysis

Objective: The goal of this lab is to introduce you to various lab and analytical skills that you’ll use throughout the year.

Overview: Your goal is to conduct an experiment that establishes a mathematical model between the drop height and rebound height of a bouncy ball (or tennis ball or whatever else that works...). This lab will guide you through all of the steps. As the year progresses, you will be responsible for handling these steps independently.

Procedure: You and your partner(s) will use the bricks on the wall as a coordinate system (assuming all of the bricks are identical in height). Thus your length-unit will be “bricks.” You will raise the ball to various heights and measure the maximum height the ball reaches after its first bounce (or rebound). You are to record the drop height and rebound height of the ball (in units of “bricks”). Since the ball itself has appreciable height, you need to be consistent in your measurement techniques.

Data: Notice that the data table has a title, is clearly labeled, and includes the units of measurement. Your data tables should always include these components and be legible.

<table>
<thead>
<tr>
<th>The Drop Height Versus Initial Rebound Height of a Bouncy Ball</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Drop Height (Bricks)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Note: You could take more or less data than what the table provides for. As the ‘head researcher’ this is your prerogative. Please note two things: 1) Reliability of your results tends to increase with the more data you collect. 2) When using linear regression (more on that below) it’s a good rule of thumb to collect a minimum of 5 data points.

Note: This lab is having you collect one data point per height. Conducting multiple trials for each height would improve the reliability of your results. We’ll discuss this more in a future lab.
**Analysis:** We will perform a lot of linear regressions this year and will do so on this lab. All of your graphs should contain the following four components: 1) A title, 2) labeled axes (including units), 3) appropriately scaled axes, and 4) accurately plotted data. If you’re doing a linear regression, you should also include 5) the line of best fit and 6) the equation that describes the line of best fit.

Using your data table and subsequent regression (see next page), ensure that your graph has the six items listed above.
Performing the Regression
Logger Pro can perform linear regressions, but we will use this software later. For this lab you could use a spreadsheet program (such as Excel) or a graphing calculator. A graphing calculator is faster and more suitable for our objectives in this lab and that is the method we will use. Excel will provide the uncertainty from your regression, but we’ll handle uncertainties next lab. Measurements without uncertainty values included are actually quite useless, but we’re conveniently ignoring this fact for one lab.

Using a TI-84 (button names are capitalized):

- If you don’t have the most updated OS (2.55 or later), then make sure to turn stat diagnostics on.
  - Hit 2nd, CATALOG, D, select DiagnosticOn and press Enter. This will cause your calculator to provide the correlation coefficient. If you have the update, the choice to turn diagnostics on should show up on your screen as you do the following steps.
- Press STAT, and choose Edit.
- Enter your x-axis data under L1 and your y-axis data under L2
  - You can enter your data under any list. This is just a convenient way of entering the data.
- Press STAT, choose Calc, and choose option 4 LINREG.
- Make sure your x-data is matched with L1 and your y-data is matched with L2. Press ENTER.
- Your calculator should provide the slope and y-intercept for the line of best fit. Record these values to one decimal place (more discussion about precision will be had in the next lab).

Making Sense of Your Linear Regression Equation
You should have graphed the initial drop height on the x-axis and the maximum rebound height on the y-axis. Your equation may look something like,

\[ y = 0.7x + 0.1 \]

I made these numbers up. Yours will likely be different.

What does this mean in the context of this lab? Let’s replace y and x with the variables they represent:

Rebound Height = 0.7(Drop Height) + 0.1.

If the drop height was 5 bricks, our equation predicts that the rebound height would be:

Rebound Height = 0.7(5) + 0.1 = 3.6 bricks.

So the rebound height is 70% of the drop height plus 1/10 of a brick. Let’s look in more detail at the slope.

Another phrase for slope is “average rate of change.” Our slope tells us how much (or “fast”) the rebound height should change as the drop height changes. The unit of our y-data is “bricks” and the unit of our x-data is “bricks.” Recall that, slope = \( \Delta y / \Delta x \). To get the units of the slope, we divide the units of y by the units of x (we treat units much like we treat algebraic variables). Thus our units on our slope are \( \frac{\text{bricks}}{\text{bricks}} \). Our slope is a ratio of the rebound height to the drop height. In other words, for every brick you increase the drop height, you would expect the rebound height to increase by 0.7 bricks.
Now what about our y-intercept? We would expect that if we dropped a ball from a height of zero that it would rebound to a height of zero – meaning we expect a y-intercept of zero. But our equation predicts a rebound height of 0.1 bricks in this situation – strange! The issue arises from the fact that there is uncertainty and error in your measurements. The linear regression is a fancy way to take a reliable average – but it is not error free! This highlights that all mathematical models have limitations in their ability to describe physical reality.

Units
Units are vital in physics! Quantities without units are meaningless! Mathematically, units behave much like algebraic variables. In Algebra, you can only add “like terms.” In Physics, you can only add quantities that have identical units. Also, saying two sides of an equation are equal in physics is tantamount to saying both sides have equal units as well!

Let’s look at our best-fit equation (I replaced y with “R” for rebound-height and x with “D” for drop-height):

\[ R = 0.7D + 0.1 \]

We’ll rewrite this equation with the units included:

\[ R \cdot \text{bricks} = \left( 0.7 \frac{\text{bricks}}{\text{bricks}} \right) (D \cdot \text{bricks}) + 0.1(\text{bricks}). \]

Using the fact that units “divide out” like algebraic variables do gives us:

\[ R \cdot \text{bricks} = 0.7 \cdot \text{bricks} + 0.1 \cdot \text{bricks}. \]

This matches what we said earlier. On the right we have “bricks” + “bricks” which is good because we can only add “like units.” On the left we also have units of “bricks.” Thus we have “bricks” = “bricks” which is good because only quantities of identical units can be equal. In Physics, we include units with all calculations, intermediate steps, and final results!

While this is a bit of a “silly” example, the skill transfers to the study of physics. We will often look at the units of slopes and intercepts to gain insight into equations.

Correlation Coefficient
To oversimplify things, we can view a correlation coefficient as a measure of how “well” a model fits the data. The coefficient is typically represented by the variable \( r \). The values of \( r \) range from -1 to 1, inclusive. The closer \(|r|\) is to 1, the “better” our model fits the data. We will mostly concern ourselves with the square of this coefficient, \( r^2 \). If \( r = 0.998 \) and \( r^2 = 0.996 \), we (roughly) conclude that 99.6% of the variance on the rebound height is due to the variance in the drop height. In other words, 0.4% of the pattern in the data is due to some other factor. This greatly oversimplifies things, and those of you who take a stats course will learn more about this later, but such a view of \( r^2 \) suits our purposes. In short, the closer \( r^2 \) is to 1, the more reliable our model is as a descriptor of our data.
Model Limitations
You will eventually be responsible for understanding the underlying assumptions of a particular model and the inherent limitations of the model.

Our model in this lab is a linear trend – holding a constant rate of change. Our model is fairly accurate for relatively small drop heights. But what if we dropped the ball from a really tall height, say 3000 bricks? Well from that height the effects of air resistance will be greater, the strength of gravity will vary an appreciable amount, and the structural integrity of the ball is likely to be permanently altered upon impact with the ground. So our model only maintains accuracy over a drop distance comparable to the data set. One key idea in all of science is that all models have underlying assumptions and limitations that affect their usefulness and accuracy.

Sources of Error
All measurements have error and uncertainty associated with them. Suppose one of your rebound heights measured 3.4 bricks. How sure are you in that measurement? Was it exactly 3.4 bricks? For example, did it rise to a height of 3.400000 bricks? Or was it 3.42 bricks? 3.35 bricks? You “eyeballed” your measurement – a highly imprecise technique.

Consider another issue affecting your measurements. Did you measure from the bottom of the ball each time? Or did you measure from the top one time and the middle the next and maybe the bottom another time? The ball is an appreciable height of one of the bricks, so consistency here matters a great deal.

In our labs, you will be responsible for identifying one or more significant sources of error and how such sources could effect the final result. You will also be responsible for identifying how the lab could be redone to minimize such error.

For this lab, let’s say you measured the drop height from the bottom of the ball but you measured the rebound of the height by watching the top of the ball. This would make all of your rebound heights erroneously large by an amount equal to the height of the ball. This would cause the slope of our line of best fit to be erroneously high and we would conclude that the ball is “bouncier” than it truly is. This is a systematic error (you made the same mistake each trial) and can be easily remedied by redoing the lab and measuring from the bottom of the ball at all times. You could also improve the accuracy by using slow-motion video analysis instead of just “eyeballing” the bounces.

We’ll discuss specific types of error in later labs.

Post-Lab
Use the information above and your brain to complete the following:

1. Complete the data table with your measurements.
2. Label the provided graph completely, plot the data points, and draw your line of best fit.
3. Perform the linear regression and record your equation and correlation coefficient below.
4. According to your best-fit equation, what is the ratio of the rebound height to the initial drop height?

5. Use your best-fit equation to predict the rebound height if the ball were dropped from a height of 20 bricks.

6. Suppose you wanted (for some reason only known to you) the ball to rebound to a height of 15 bricks. What initial height would you have to drop your ball from?

7. Suppose the person dropping the ball were accidentally giving the ball a small downward push. How would this error effect your results?

8. Does a slope greater than 1 seem plausible in this lab? Explain.

9. Suppose a slope greater than 1 is obtained from this lab. Explain a reasonable error that could have lead to this result.

10. Does a negative slope seem plausible in this lab? Explain.
### Rubric

<table>
<thead>
<tr>
<th>The procedure and data collection process were properly followed.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appropriately titled, labeled, and scaled graph with correctly plotted data and reasonable LOBF drawn in.</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Correct equation describing the LOBF is included with the $r^2$-value.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Correct use of the LOBF in the context of the lab is clearly demonstrated {Qs 4-6}</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Demonstration of a clear understanding between the concepts and mathematics involved in the lab {Qs 7-10}</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Total Points out of 35:</td>
<td>General comments:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>